

ARISTOTLE AND MATHEMATICS

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I am deeply honored to be invited to speak at the inauguration of the Interdisciplinary Center for Aristotelian Studies at the Aristotelian University of Thessaloniki. My sincere thanks to Professor Demetra Sfendoni-Menzou for inviting me to participate in this event as well as for playing so important a role in the foundation of the Center. On a more personal note I want to say that I am delighted that my daughter is here in the room -- I am proud that she is a student in the Architecture School here at the university.

Before talking about mathematics I want to say a word about interdisciplinarity. I cannot stress how important it is to employ an interdisciplinary approach in studying Aristotle. In my mind, the first discipline needed for any proper work in Aristotle is philology -- a knowledge of ancient Greek and in particular of Aristotle's vocabulary, his technical terminology, his syntax, and his style, as well as the unusual history of his text and of the textual tradition. Without this knowledge one is dependent on translations, and anyone who has worked in Aristotle knows how much translations of the same passage can differ from one another. In many cases the differing translations cannot all be correct, and some cases they may well all be wrong. So the first requirement is to be able to work with the original text.

Next, as when working with any ancient author, one needs to be familiar with related authors and texts, in the present case, principally the works of Aristotle's philosophical predecessors, but in some cases other texts as well. Students of the *Poetics* need to be familiar with tragedy, comedy and epic, for example, especially the *Oedipus Tyrannos* and the *Iphigeneia en Taurois*, to which Aristotle frequently refers. Students of Aristotle's biological works should be familiar with earlier work on biological and medical subjects. Students of Aristotle's *Physica*, *Peri Ouranou*, *Peri Geneseos kai Phthoras*,

Meteorologica, and other scientific works should know about earlier treatments of the same subjects, primarily in Plato and the Presocratic philosophers.

In most cases it is also useful to know something about the later history of the subject up to and including current (twenty-first century) ideas on the Aristotelian field one is investigating. As Professor Sfondoni has shown in many of her publications and many of the conferences she has organized, "Aristotle Today" is a lively topic, and I expect that it will be a focus of much of the work of the new Center.

I would also point out that in an important sense, it was Aristotle who invented the notion of an intellectual discipline, without which it would be impossible to speak of interdisciplinarity. I follow Aristotle in defining a discipline as a field of study marked off from other disciplines by its subject matter. Briefly, before Aristotle, the area that we call *thetikê epistêmê* was not differentiated into different fields. A single writing would cover topics in the theory of matter, physics, cosmology, astronomy, meteorology, psychology and biology. Such works tended to be known by the general title "*Peri Phuseos*," where "*phusis*" covered the entire range of natural phenomena. Aristotle changed this, and he changed it on two levels. First, he had so much to say about so many topics that he devoted entire works to individual subjects, as titles like *Peri Ouranou* and *Meteorologica* indicate. But he backed up this division of knowledge through an elaborate theory of *epistêmê*, set out in the *Analutica Ustera*. The *Analutica Ustera* is a difficult work in many ways, but even so it has had an important role in the history of scientific thought. Perhaps surprisingly, it is not placed among Aristotle's scientific works, but in the series of treatises known as the *Organon*. These are treatises on logic.

The *Analutica Ustera* tells us that each science has a distinct field of subjects that it studies. Aristotle calls this the *upokeimenon genos*, and he gives examples. Arithmetic studies numbers, geometry studies *megethê*. Note that these are examples taken from mathematics. Each *genos* consists of things that naturally go together. Geometry, for example, studies lines, circles, triangles, and so forth.

More specifically, a science investigates the properties of its subjects. Aristotle's favorite example is that every triangle has angles equal to two right angles. Here the subject is 'triangle' and the attribute is 'having angles to two right angles.' In school we learned how to prove this result, and Aristotle knew a proof that was exactly the same as the one we learned, or one that was very close to it. He took geometry as his model for all sciences. As in geometry, every science proves its conclusions, that is, each proof in a science proves that a subject in the subject genus of that science possesses an attribute. Sometimes Aristotle speaks as if the attributes also are part of the subject genus. From that point of view, a science will establish all the provable connections between subjects and attributes in its subject genus.

If we remember our geometry well enough, we remember that not all geometrical facts are proved. In fact, it would be impossible to prove all of them, as Aristotle shows. The most basic facts are stated at the beginning. In Euclid's geometry, there are three kinds of basic facts: definitions, *aitêmata*, and *koines ennoies* (sometimes called *axiômata*). The purpose of the definitions is to specify precisely what we are talking about. For example, the definition of an isosceles triangle as a triangle that has two equal sides makes it clear what triangles are isosceles. There is no way to prove this definition any more than there is a way to prove that a football match lasts 90 minutes. It is just a basic fact. And from these basic facts geometry goes on to prove other facts, for example, that the angles opposite the equal sides in an isosceles triangle are equal.

Aristotle took geometry as a model for other sciences. After all, geometry had made an astonishing amount of progress because of the way it proved its results. Geometers agreed on the principles. They also agreed about how proofs work and what proofs are successful. As a result, they had to agree on the conclusions of their proofs. In geometry there was a consensus about facts and methods. In *arithmetikê* (that is, what we call number theory) there was a similar consensus, and likewise in other branches of mathematics, stereometry, for example. This success was unmatched in other scientific

fields. In them there was no agreement about the basic entities: were they the four elements (fire, air, water and earth) as Plato said, following Empedocles, or were they atoms and void as Democritus said, or were they limiters and unlimiteds as Philolaus said? Further it seems unlikely that there was agreement about the scope of their study (what Aristotle calls the subject genus) or about the methods of establishing, confirming or proving results.

The *Analutika Ustera* prescribes that each science (*epistêmê*), like Euclid's geometry, has three kinds of principles: definitions (the same as in Euclid), hypotheses (which correspond roughly to Euclid's postulates) and common principles or axioms (which correspond in part to Euclid's axioms). It prescribes acceptable proof techniques. And it specifies that when we know something through a proof we know both that it is true and why it is true. It is true because it follows from the basic facts of the science, which are stated in the principles.

There is clearly a close connection between Aristotle's view of science and Euclid's geometry. We may wonder whether Aristotle got his ideas from geometry or Euclid got his ideas from Aristotle. There is not time here to discuss this matter, but there is good reason to think that the answer is "both of the above" -- that Aristotle took over the idea of proof and of the role of definitions and axioms in geometrical proofs. Possibly postulates as well. He considered from the standpoint of logic and metaphysics how proofs work and what they can and cannot prove, and he generalized the notion of proof to work not just for geometry but for any scientific field. In turn, Euclid's geometry seems indebted to Aristotle for several features that make little sense from the point of view of geometry but which he seems to have included in order to follow Aristotle's prescriptions as closely as possible.

The theory presented in the *Analutika Ustera* was the first general theory of scientific explanation ever worked out in detail, and the rigor it demanded remains with us today as a scientific ideal. This is true even though proof techniques have changed dramatically, especially since the time of Galileo (who, by the way, before he rejected

Aristotelian physics, made a living by giving lectures on the *Analutika Ustera* among other works of Aristotle).

This is one important connection between Aristotle and mathematics. But, to return briefly to the theme of interdisciplinarity, the *Analutika Ustera* also speaks specifically about certain kinds of interdisciplinary work. I am thinking of sciences that are closely related in certain ways. Take optics. In antiquity optics was not the same as it is for us. Today optics is defined as the branch of physics which investigates the behavior and properties of light whereas Aristotle defined optics as the science concerned with lines “in sight” (*en opsei*), where lines are among the subjects studied by geometry. Now this account seems problematic. According to Aristotle’s doctrine of the subject genus, if lines are part of the subject genus of geometry, they cannot be part of the subject genus of a different science; and yet he maintains that optics is a different science from geometry. Fortunately we possess another work of Euclid’s that helps us understand what Aristotle means. This work is called *Optics*. In Euclid’s *Optics* we find that optics considers how things appear to us; it uses proofs, and it makes use of geometry in those proofs. For example, optics proves that something (A) appears larger than something else (B) by first showing that A appears under a larger angle than B and then invoking the principle that things seen under larger angles appear larger. The first part of the proof is pure geometry, beginning with geometrical definitions and proving a geometrical theorem; the second part depends on a “bridge principle” which relates a geometrical property (one angle is larger than another angle) to the subject matter of optics (things seen under larger angles appear larger). The reference to how things appear has no place in geometry, but it does in optics, which is the science of how things appear. Aristotle describes the relation between these sciences as follows: optics is subordinate to geometry and conversely geometry is superior to optics, in the sense that optics knows the fact (the fact that things seen under larger angles appear larger) and geometry provides the explanation. In some sense, optics is applied geometry. It applies purely geometrical facts to its special subject matter. But it

qualifies as a separate science since its subject matter is different, and it has distinct principles (such as the bridge principle used in the sample proof) that apply to optical objects, not geometrical objects. Most of Aristotle's examples involve pairs of sciences, but occasionally he speaks of cases where three sciences fall one under the other. And he once says that in a small way medicine is subordinate to geometry, because the doctor knows that circular wounds heal slowly while geometry provides the explanation. So here we have cases where interdisciplinarity is needed. In order to do optics, one needs to know geometry. In order to do be a specialist in optics it is not necessary to be a specialist in geometry, or even be able to understand fully all the proofs that geometry contains. But it is necessary to be familiar with the principles of geometry and with the theorems that are relevant to the properties that optics studies. So it appears that Aristotle was not only the father of disciplinarity, he was also the father of interdisciplinarity, again making precise determinations of what kind of interdisciplinarity he recognizes and of the role played by each of the contributing disciplines.

I will return now to mathematics. We have seen that Aristotle used mathematical practice as a model for other sciences and that he proposed a radical reform in the other branches of knowledge which, if it were successfully accomplished, would raise them to the rank of true *epistêmê*. I have proposed that Aristotle did this not by making mathematical discoveries of his own, but by reflecting on the nature of mathematics. In other words, by being a philosopher, not by being a mathematician.

In fact, Aristotle was not a mathematician. There are theorems named after Thales and Pythagoras, but none named after Aristotle. According to Darwin, Aristotle was the greatest biologist who ever lived. According to John Rawls, the Harvard professor whose work *A Theory of Justice* is widely recognized as one of the foremost works in the areas of social, political and moral philosophy in the second half of the 20th century, Aristotle's principal work on ethics, *ta Êthika Nikomacheia*, is the best book on moral philosophy that has ever been written. He was the inventor of logic, and his theory of the syllogism is still

taught to this day in certain universities. His account of the physical world, in combination with Ptolemy's astronomy held sway for over a millenium and it took the combined genius of Galileo, Kepler and Newton to replace it. But mathematics he left to others.

Nevertheless, he was aware of what was going on in mathematics and he had views on mathematical subjects. There are many references to mathematics in his works. This is a large subject. Thomas Heath, the author of the principal English-language work covering the entire history of Greek mathematics, on his death left behind translations and commentary on the Aristotelian passages relevant to mathematics. To publish this material was evidently a passion of his. After his death his widow wrote that "his eagerness [to complete] this work ... was probably instrumental in hastening his end." The 291 page book was published in 1949 with the title "Mathematics in Aristotle" and it remains one of the principal sources of information on this topic. I have time for only a few examples.

First, Aristotle was up to date at least in some areas of mathematics and understood the technical aspects well enough to voice views. For example, he noted a problem in the theory of parallel lines as it was known in his time, and the need to solve this problem may well be the reason why Euclid introduced two additional postulates into geometry..

Aristotle also drew general conclusions on the basis of what goes on in mathematics. For example, like all triangles, isosceles triangles have angles equal to two right angles, and it is possible to prove this. But it's not enough just to get a correct result, it is important to reach the result in the correct way. Aristotle maintains that it is incorrect to prove this particular result directly; instead we should prove that all triangles have angles equal to two right angles -- which implies immediately that isosceles triangles do.. This example is used to show that proofs should be as general and powerful as possible, not only because it simplifies the work of a science but also because it also puts things in the right order. Isosceles triangles have the property in question because they are triangles. If you have a separate proof that they have that property, you do not understand this essential fact. Your

understanding of the subject genus is faulty -- in other words, you don't really know what you are talking about.

The last contribution that I will mention is Aristotle's sophisticated conception of the infinite. The infinite is a concept that is needed in mathematics and in physics. Numbers can be increased *ep' apeiron*; whatever number you take, there is always a number larger than it. Likewise, continuous lines, distances, and times can be divided *ep' apeiron*. You can always divide a line, a distance, or an interval of time in two: if you have a line one meter long you can divide it into two lines half a meter long each; if you have a time interval one minute long you can divide it into two intervals, half a minute long each. And the same holds for the half-meter lines and the half-minute intervals. However far you divide them there are smaller lines and time intervals. Earlier thinkers had discussed the infinite. Zeno in particular had exploited it in order to prove his controversial view that motion is impossible. But for Aristotle, motion is at the heart of physics, since he conceives the objects of physics as things that can undergo motion. This made it imperative for him to refute Zeno's arguments, and in order to do so he needed to get clear on the nature of continuity and of the infinite. He did important work in this direction in *Physics*, not only refuting Zeno's arguments to his satisfaction, but recognizing that the problem of the infinite goes deeper than Zeno realized, and then setting out a theory of the nature of the infinite which, although quite different from today's notion of mathematical infinity, was consistent with his physics, was grounded in his metaphysical distinction between actuality and potentiality, and was the prevalent view (even among mathematicians) until the nineteenth century.

As I have indicated, there is more to say on the subject of Aristotle and mathematics. But I hope that these brief remarks have given you an idea of how important mathematics was for Aristotle, how his reflection on mathematical topics was an important factor in some of his most important philosophical ideas, and in turn how these ideas helped shape

mathematical and scientific thought for centuries and in some cases for millenia afterwards.